CALCULATION OF THE STRESS–STRAIN STATE OF BENT ANNULAR PLATES TAKING ACCOUNT OF MATERIAL DAMAGE DURING CREEP

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Sets of equations used which describe material creep with simultaneous consideration of damage accumulation drawing on equilibrium equations, the compatibility of strains and the corresponding boundary conditions, have made it possible to unite into one problem two independent problems [the problem of determining the stress–strain state (SSS) for an arbitrary body (structural element) loaded by known external loads and temperature, and the problem of determining the time for the start of its failure] [1]. Solution of the set of equations obtained is connected with certain mathematical difficulties [1] and it requires (even in the case of simple structural elements) considerable computer time [2]. In view of this on the basis of the mixed variation principle of MacComb, Plekhte, and Sanders an approximation method for calculating the SSS of structural elements and the time for the start of their failure is suggested in [3-5]. The method makes it possible to reduce the problem in question to a similar problem assuming secondary creep for the material. In order to obtain the result sought it is necessary to multiply the known solution of the secondary creep problem by functions of coordinates and time for which approximate ratios and also upper and lower limits are obtained.

Using this method the problem is solved below for calculating the SSS and time for the start of failure for bent uniformly heated annular plates. The results are compared with similar results obtained by the traditional method of calculation in steps with respect to time [1].

Differential equilibrium equations for an annular plate element are written in the form [6]

$$\frac{\partial (Qr)}{\partial r} = pr, \frac{\partial (M_r r)}{\partial r} - M_\varphi = -Qr, \tag{1}$$

where r is current radius; Q is intensity of transverse force in the circumferential section of the plate; M_φ, M_r are intensities of bending moments in circumferential and radial sections, respectively; and p is intensity of load distributed at the plate surface. The intensity of bending moments is connected with radial and circumferential stresses by the relationships [6]

$$M_r = \int\limits_{-h/2}^{h/2} \sigma_r z\, dz,\ M_\varphi = \int\limits_{-h/2}^{h/2} \sigma_\varphi z\, dz. \tag{2}$$

Here z is distance from the central plane of the plate to the current section; h is plate thickness [h = h(r)].

For simplicity we limit ourselves to calculation for a plate without considering elastic strains. In this case the rate of radial η_r and circumferential η_φ creep strains will be connected with the rate of the angle of rotation for the normal to the central surface of the plate ψ by the relationships [6]

$$\eta_r = z\frac{\partial \psi}{\partial r}, \quad \eta_\varphi = z\frac{\psi}{r}. \tag{3}$$

The set of equations which describe material creep with simultaneous consideration of its damage are presented as [3, 4]

$$\eta_r = B_1 S_2^{(n-1)/2}\frac{2\sigma_r - \sigma_\varphi}{6\mu^m},\ \eta_\varphi = B_1 S_2^{(n-1)/2}\frac{2\sigma_\varphi - \sigma_r}{6\mu^m},$$

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$$\mu = \left[1 - (m+1) B_2 \int_0^t S_2^{(g+1)/2} d\tau\right]^{1/(m+1)}, \tag{4}$$

where $S_2 = (\sigma_r^2 + \sigma_\varphi^2 - \sigma_r\sigma_\varphi)/3$ is tangential stress intensity; B_1, B_2, m, n, g are material characteristics; μ is the damage parameter.

The set of equations (1)-(4) together with boundary conditions make it possible to calculate the SSS for bent plates at any instant of time up to the start of their failure.

According to the approximation method [3-5] the SSS of plates will be found in the form

$$\begin{gathered}\sigma_r = \sigma_r^0 \mu^{m/n}/X(t), \sigma_\varphi = \sigma_\varphi^0 \mu^{m/n}/X(t), \\ \eta_r = \eta_r^0 F(t), \ \eta_\varphi = \eta_\varphi^0 F(t).\end{gathered} \tag{5}$$

Here functions X(t), F(t), μ(r, z, t) are determined approximately by the expressions

$$X = (1 - t/\bar{t}_*^0)^\gamma, \quad F = X^{-n}; \tag{6}$$

$$\mu^{m/n} = \left\{1 + \frac{\bar{t}_*^0}{t_*^0}\left[\left(1 - \frac{t}{\bar{t}_*^0}\right)^\nu - 1\right]\right\}^\beta. \tag{7}$$

Here

$$t_*^0 = \left[(m+1) B_2 S_2^{0(g+1)/2}\right]^{-1}; \ \bar{t}_*^0 = \int_V W^0 dV \Big/ \int_V (W^0/t_*^0)\, dV;$$

$$W^0 = \sigma_{ij}^0 \eta_{ij}^0; \ \gamma = m/[n(m+1)]; \ \beta\nu = \gamma;$$

$$\beta = \frac{m}{n + m(n-g-1)}; \quad \nu = \frac{n + m(n-g-1)}{n(m+1)};$$

components $\sigma_r^0, \sigma_\varphi^0, \eta_r^0, \eta_\varphi^0$ are the solution of a similar problem assuming secondary creep obtained from (1)-(3) with the condition that creep strain rate components are connected with stress components by the relationships [1]

$$\eta_r^0 = B_1 S_2^{0(n-1)/2} \frac{2\sigma_r^0 - \sigma_\varphi^0}{6}, \ \eta_\varphi^0 = B_1 S_2^{0(n-1)/2} \frac{2\sigma_\varphi^0 - \sigma_r^0}{6}. \tag{8}$$

By using (2), (3), and (5) we express $\sigma_r^0, \sigma_\varphi^0, S_2^0$ in terms of the intensity of bending moments M_r^0, M_φ^0. After normal transformations we have [6]

$$\sigma_r^0 = bM_r^0 z^{1/n}, \ \sigma_\varphi^0 = bM_\varphi^0 z^{1/n}, \ S_2^0 = b^2 M_2^0 z^{2/n}, \tag{9}$$

where $b = (2n+1)(2/h)^{(2n+1)/n}/2n$; $M_2^0 = (M_r^{0^2} + M_\varphi^{0^2} - M_r^0 M_\varphi^0)/3$. From (8) and (9) we obtain

$$\begin{gathered}\eta_r^0 = B_1 M_2^{0(n-1)/2} b^n z (2M_r^0 - M_\varphi^0)/6, \\ \eta_\varphi^0 = B_1 M_2^{0(n-1)/2} b^n z (2M_\varphi^0 - M_r^0)/6.\end{gathered} \tag{10}$$

Now taking account of (9) and (10) t_*^0 and $\bar{t}_*^0$ in (6) and (7) are written in the form

$$t_*^0 = \left[(m+1) B_2 b^{g+1} z^{(g+1)/n} M_2^{0(g+1)/2}\right]^{-1},$$

$$\bar{t}_*^0 = \frac{(2n+g+2)(2n)^{g+1} \int_{r_1}^{r_2} r M_2^{0(n+1)/2} (2/h)^{2n+1} dr}{B_2 (m+1)(2n+1)^{g+2} \int_{r_1}^{r_2} r M_2^{0(n+g+2)/2} (2/h)^{2n+2g+1} dr}.$$

Using (2), (5), and (9) we obtain for M_2 at any instant of time up to the start of failure an expression

$$M_2 = M_2^0 b^2 \left(\int_{-h/2}^{h/2} z^{(n+1)/2} \mu^{m/n} dz\right)^2 \Big/ X^2.$$

The time for the start of plate failure is calculated from (7) with the condition that at the instant of failure $\mu(t_*) = 0$ [3, 4], and the angle of rotation of the normal to the central surface is calculated from the relationships (3), (5), (6)-(10).

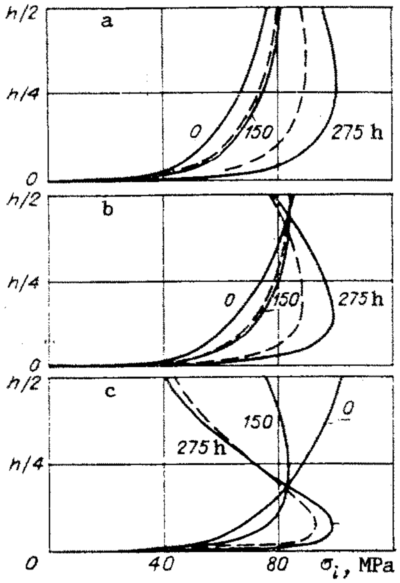

Fig. 1

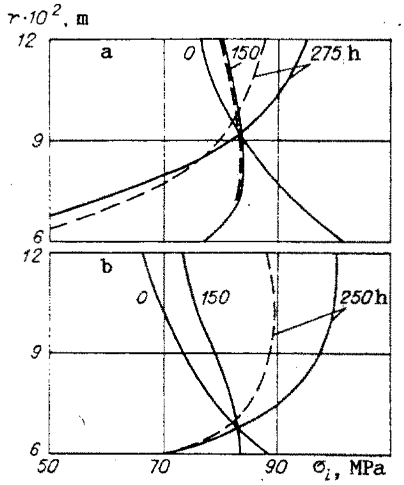

Fig. 2

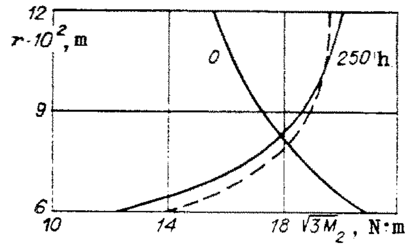

Fig. 3

Numerical calculations were carried out for annular plates of constant and variable thickness. Here the material characteristics are as follows: $n = 5$, $g = 5$, $B_1 = 0.379 \cdot 10^{-12}$ $MPa^{-n} \cdot h^{-1}$, $B_2 = 0.252 \cdot 10^{-13}$ $MPa^{-(g+1)} \cdot h^{-1}$, $m = 10$. The first example considered was an annular plate with radius $r_2 = 0.12$ m with a hole of radius $r_1 = 0.06$ m of constant thickness $h = 0.03$ m loaded by a bending moment $M_0 = -14$ N·m distributed uniformly over the inner contour. Here the outer contour was assumed free from load. Boundary conditions took the form

$$M_r(r_1) = M_0, \quad M_r(r_2) = 0.$$

The solid lines in Fig. 1 are the distribution of stress intensity over the height of the plate at the outer ($r = r_2$), central [$r = (r_1 + r_2)/2$], and inner ($r = r_1$) contours (a-c, respectively), and in Fig. 2 over the plate radius in sections $z = h/4$ and $z = h/2$ (a, b) determined at different instants of time using the approximation method. Here broken lines show the similar distribution obtained by calculation in steps with respect to time. Shown in Fig. 3 is the distribution of $\sqrt{M_2}$ over the plate radius at different instants of time obtained by the approximation method (solid lines) and with calculation in steps with respect to time (broken lines).

In the second example an annular plate was considered with radius $r_2 = 0.12$ m with a hole $r_1 = 0.07$ m with variable thickness loaded by a uniformly distributed load over the outer contour with bending moment $M_0 = 8.5$ N·m. The inner contour was assumed to be free from load. Boundary conditions in this case are as follows:

$$M_r(r_1) = 0, \quad M_r(r_2) = M_0.$$

The plate profile is shown in Fig. 4. Presented in Fig. 5 is the distribution of stress intensity over the plate height at the outer ($r = r_2$), central [$r = (r_1 + r_2)/2$], and inner ($r = r_1$) contours (a-c, respectively) obtained by the approximation method (solid lines) and with calculation in steps with respect to time (broken lines). Shown in Fig. 6 is the change in $\sqrt{M_2}$ over the plate radius at different instants of time (solid lines are the approximation method, broken lines are numerical calculation in steps with respect to time).

A reliability criterion for the approximation method may be comparison of the time for the start of failure with the similar time obtained by numerical calculation in steps with respect to time (of course it is possible to limit oneself to comparing stress fields at different instants of time). In the first problem $t_* = 287$ h obtained by the approximation method, and by the numerical method $t_* = 275$ h, and in the second problem $t_* = 276$ and 270 h.

Calculations were performed in an ES-1022 computer. The machine time used in solving the problem was 2.5 min by the approximation method, and 2 h by the calculation in steps with respect to time.

On the basis of this it is evident that the method in question may be recommended for use in practice since it gives entirely suitable results much more simply and effectively

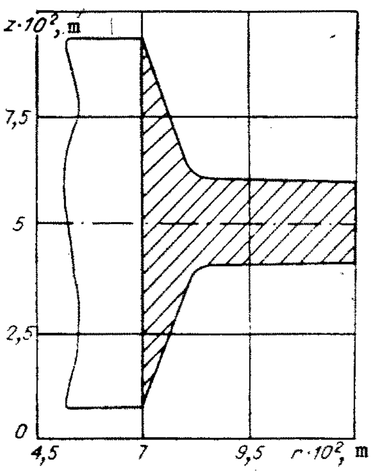

Fig. 4

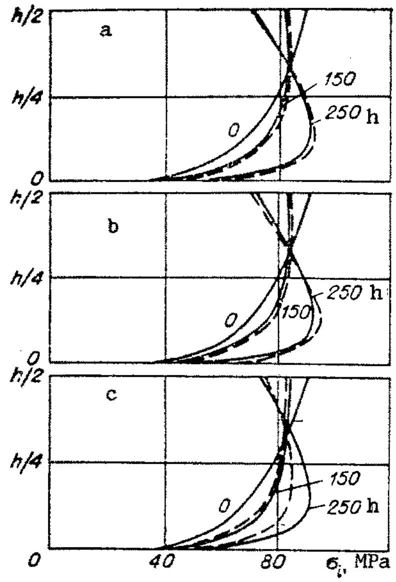

Fig. 5

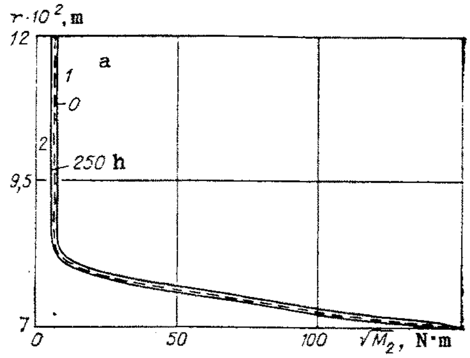

Fig. 6

compared with the traditional calculation in steps with respect to time and it requires the minimum use of a computer.

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